EMa2 Linear Systems & Partial Differential Equations

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WARNING: THESE ARE NOT A COMPLETE SET OF NOTES. YOU WILL NEED TO ATTEND THE LECTURES TO COPY FURTHER ANNOTATION TO THE NOTES, AND THE WORKINGS TO EXAMPLES.

0. Overview

6 topics, 6 weeks, 3 lectures a week

1. **Fourier series** (3 lectures) What is the difference between a periodic signal and a sinusoidal signal? Periodic signals and functions. Odd and even functions. How sinusoidal is a signal? Expressing periodic functions in terms of sines and cosines. Lots of worked examples. The Gibbs phenomenon. Half-range sine and cosine series.

2. **Fourier transforms** (2 lectures) How do we decompose a signal into its frequency components? Basic properties. The time domain and the frequency domain. Frequency transfer functions and frequency response functions. What is the `fft`?

3. **Laplace transforms** (3 lectures) Extracting the decay rate from signals. Basic properties. Transforms of common functions and the inverse transform. Using Laplace transforms to solve ODEs. The transfer function.

4. **Introduction to PDEs** (2 lectures) What are PDEs? Are they just ODEs grown up? Why are they useful in Engineering? What does it mean to *solve* them? The importance of boundary conditions. 3 great equations: heat, wave and Laplace equations. 2nd-order linear constant coefficient PDEs. Hyperbolic, elliptic and parabolic.

5. **The Separation of variables** (5 lectures) A 'suck it and see' technique. Satisfying the boundary conditions Solution process for heat, Laplace and wave equations (the first 2 steps). But how to find the unknown coefficients of the periodic functions? The use of Fourier series to solve for the unknown boundary condition. The principle of linear superposition and how to deal with inhomogeneous equations.

6. **Characteristics and d'Alembert's method** (2 lectures) d'Alembert's method for the wave equation on an infinite domain. Examples. General solution. d'Alembert's method for semi-infinite domain.

HINT: *Don't panic!* Lots of worked examples in the lecture. Only 5 examinable techniques: Fourier series, Fourier transforms, Laplace transforms, separation of variables and d'Alembert's method.

BUT you must be *au fait* with one-dimensional calculus, especially

- Integration by parts
- Partial fractions
- Solution of 2nd-order ODEs.

REVISE THESE TOPICS NOW!