**Q4** (a) A Poisson distribution satisfies $P(X=1)=P(X=3)$. Determine its mean and the common probability.

*(3 marks)*

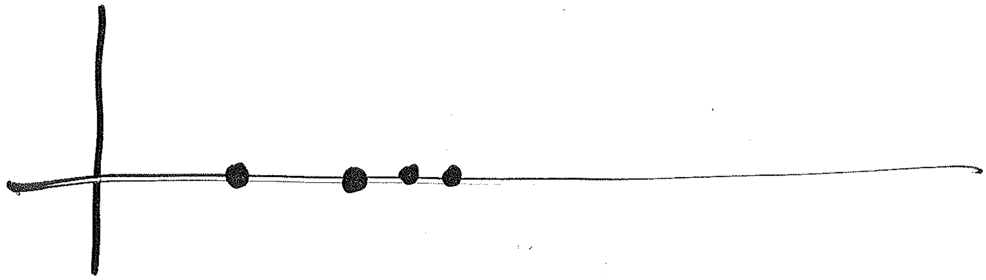
(b) A radioactive source emits 500 rays every second. Determine

(i) The probability that exactly 3 rays are emitted in 6 milliseconds.

(ii) The probability that 2 or more rays are emitted in 4 milliseconds.

(iii) The probability that 2 or more rays will be emitted in 4 milliseconds if 1 ray is already known to have been emitted.

*(7 marks)*

(a) 

$$P(X=x) = \frac{e^{-\mu}\mu^{x}}{x!} \; ; x = 0, 1, 2 \ldots$$

($\mu$ mean)

$$P(X=1) = \frac{e^{-\mu}\cdot\mu}{1} = \mu e^{-\mu}. \quad =$$

$$P(X=3) = \frac{e^{-\mu}\cdot\mu^{3}}{3!} = \frac{\mu^{3}e^{-\mu}}{6}$$

$$\frac{\mu^{2}}{6} = 1, \text{ so } \mu = \sqrt{6} \doteqdot 2.45.$$

$$\text{Prob} = \sqrt{6}\cdot e^{-\sqrt{6}} = 0.211.$$

(b)(i) $\lambda = \cancel{\frac{1}{500} = .002}$. $t = 6\times10^{-3}$.

$\cancel{\lambda t = 0.002 \times 6\times10^{-3} =}$

$\lambda = 500$, $t = 6\times10^{-3}$, so $\lambda t = 3$.

so $\mu = 3$. $P(X=3) = \frac{e^{-3}\cdot 3^{3}}{3!} = 0.224$.

(ii) $\lambda = 500$, $t = 4\times10^{-3}$, so $\lambda t = 2$.

$P(X \geq 2) = 1 - \{P(X=0) + P(X=1)$

$P(X=0) = e^{-2} \doteq 0.135.$

$P(X=1) = 2e^{-2} = 0.270.$

so $P(X \geq 2) = (1 - .135 - .270)$

$= 0.595.$

(iii) $E: X \geq 2.$ $F$: $x \neq 0.$ ~~0,1?~~

$P(E/F) = P(E \cap F)/P(F)$

$= P(E)/P(F)$; $E \subseteq F.$

$\cancel{= \{1 - P(x=0)\}/0.595}$

$= 0.595/\{1 - P(x=0)\}$

$= 0.595/0.865 = 0.687.$

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111, 011, 101, 110 | 010, 100, 001, 000

'1'      '0'

'p'

p correct.

1-p incorrect.

$p^3$    $p^2(1-p)$

Correct: $p^3 + 3p^2(1-p) =$

$p = 0.99.$    $0.95.$

$(0.99)^3 + 3(0.99)^2(0.01) =$

$=$

## SECTION A

**Q1** A primitive binary receiver has probability $p$ of making an error in receiving each bit of a transmitted message, i.e. there is a probability $p$ that a '1' will be received as a '0', and vice-versa. To overcome this '111' is transmitted instead of '1', and '000' instead of '0'. The triple bits '011', '101', '110', as received, are interpreted as '1', and '001', '010', '100' as '0'.

(a) Show that the probability that a triple bit transmission is correctly received is:

$$F(p) = (1-p)^3 + 3p(1-p)^2$$

How does this accuracy compare for $p = 0.01$ and 0.05.

(b) Explain why a double bit transmission, e.g. '11' for '1' for accuracy improvement, would be totally useless.

(*8 marks*)

$p = 0.01$ — error.

111 101 110 011 — Correct.

1

$(1-p)^3$

$p(1-p)^2$

$$F(p) = (1-p)^3 + 3p(1-p)^2.$$

put $p = 0.01$, so $1-p = 0.99$.

$$F(0.01) = (0.99)^3 + 3(0.99)^2(0.01)$$
$$= 0.9997.$$

---

2 bits: 11, 10, 01, 00

ok ? not ok

**Q9** (a) A light-tube has a mean lifetime of 1500h and standard deviation of 150h. Three tubes are connected so that when one tube burns out another will go on. Assuming that the lifetimes are independent and Normally distributed, what is the probability that lighting will take place for

(i) at least 5000h

(ii) at most 4200h

*(6 marks)*

(b) In a copper wire, the normal stress on a specimen is known to be functionally related to the shear resistance. The following experimental data were recorded:

| Normal stress, $x$ | Shear resistance, $y$ |
|---|---|
| 27.4 | 21.4 |
| 26.8 | 26.5 |
| 25.6 | 24.9 |
| 25.4 | 27.3 |
| 28.9 | 24.2 |
| 23.6 | 27.1 |
| 27.7 | 23.6 |
| 23.9 | 25.9 |
| 24.7 | 26.3 |
| 28.1 | 22.5 |
| 26.9 | 21.7 |
| 22.6 | 25.8 |

(i) Calculate the correlation coefficient $r$.

(ii) Using the test statistic

$$z = \frac{\sqrt{n-3}}{2}\left[\ln\left(\frac{1+r}{1-r}\right) - \ln\left(\frac{1+\rho}{1-\rho}\right)\right]$$

test the hypothesis $H_0 : \rho = -0.5$ versus $H_1 : \rho \neq -0.5$ using a 0.05 level of significance, where $\rho$ is the theoretical correlation coefficient of the underlying bivariate distribution.

*(14 marks)*

Q9. a. Assume that the lifetime of each tube is $L_i \sim N(\mu, \sigma^2)$

Then $\sum_{i=1}^{n} L_i \sim N(n\mu, n\sigma^2)$

In this case $n = 3$ and if $L = L_1 + L_2 + L_3$,

$$L \sim N(3\mu, 3\sigma^2) = N(\mu_L, \sigma_L^2)$$

So $\mu_L = 3 \times 1500 = 4500h$.

$\sigma_L^2 = 3 \times 150^2$, $\therefore \sigma_L \doteqdot 260h$.

So $P(L > 5000) = P\left(Z > \frac{5000 - 4500}{260}\right)$

$= P(Z > 1.92)$

$= 0.0274$

Also $P(L < 4200) = P(Z < -1.15)$

$= 0.1251$

Q. 9. b. Specimen test : normal stress. vs. shear resistance

| $x_i$ | $y_i$ | $(x_i-\bar{x})$ | $(y_i-\bar{y})$ | $(x_i-\bar{x})(y_i-\bar{y})$ | $(x_i-\bar{x})^2$ | $(y_i-\bar{y})^2$ |
|---|---|---|---|---|---|---|
| 27.4 | 21.4 | | | | | |
| 26.8 | 26.5 | | | | | |
| 25.6 | 24.9 | | | | | |
| 25.4 | 27.3 | | | | | |
| 28.9 | 24.2 | | | | | |
| 23.6 | 27.1 | | | | | |
| 27.7 | 23.6 | | | | | |
| 23.9 | 25.9 | | | | | |
| 24.7 | 26.3 | | | | | |
| 28.1 | 22.5 | | | | | |
| 26.9 | 21.7 | | | | | |
| 22.6 | 25.8 | | | | | |
| 311.6 | 297.2 | 0 | 0 | -29.533 | 43.045 | 47.146 |

$\bar{x} = 25.967, \quad \bar{y} = 24.767$

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\left\{ \sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2 \right\}^{1/2}} = \frac{-29.533}{\left\{ 43.045 \times 47.146 \right\}^{1/2}} = -0.6556$$

To test $H_0 : \rho_0 = -0.5$ vs. $H_1 : \rho_0 \neq -0.5$ at the 5% level

$$Z = \frac{\sqrt{n-3}}{2} \left[ \ln\left(\frac{1+r}{1-r}\right) - \ln\left(\frac{1+\rho_0}{1-\rho_0}\right) \right]$$

$$= 1.5 \ln\left(\frac{0.2080}{0.3333}\right) = -0.7074$$

This satisfies $|Z| \leq 1.96$, so accept $H_0$.