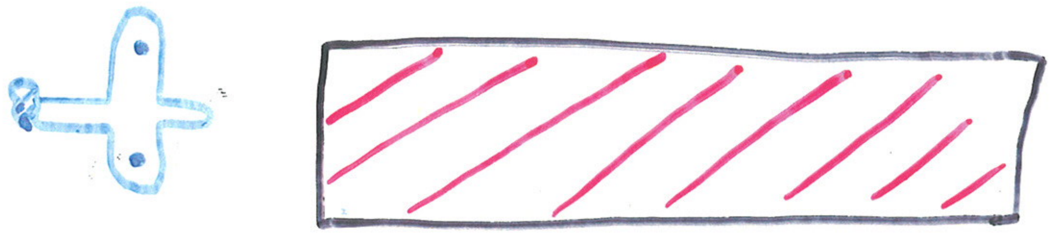
Q1.a. Search sweeps over the sea are usually rectangular.



$P(\text{success}) = 0.2. \quad q = 0.8$

If X denotes the number of sightings, and n the number of sweeps (trials), the we require $P(X \geq 2)$ in n trials.

For n ~~sightings~~ sweeps $P(X=0) = (0.8)^n$

$$P(X=1) = n(0.8)^{n-1}(0.2)$$

$$\therefore P(X<2) = (0.8)^{n-1}(0.8 + n \times 0.2)$$

For $n = 18$, we have

$$(0.8)^{17} \times 4.4 = 0.09908$$

and $n = 17$,

$$(0.8)^{16} \times 4.2 = 0.11822$$

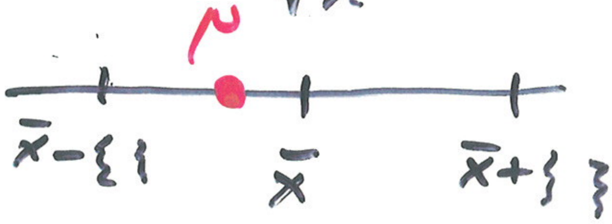
So $n = 18$ is needed to ensure

$$P(X<2) \leq 0.1.$$

Q1 (b)

$$\left| \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \right| \le 1.96 \text{ with } 95\% \text{ confidence}$$

$$\therefore \quad \bar{x} - \frac{1.96\sigma}{\sqrt{n}} \le \mu \le \bar{x} + \frac{1.96\sigma}{\sqrt{n}}$$



The window width is $O(n^{-1/2})$

To obtain 10 times greater accuracy for the window width, 100 times the sample size is needed.

i.e. $n = 39{,}000$.

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why $n = 390$?

$$\frac{1.96\sigma}{\sqrt{n}} \sim 0.1\sigma$$

$$\text{so} \quad \sqrt{n} \simeq \frac{1.96}{0.1} = 19.6.$$

$$n \sim 384 \ !$$

Q. 8

a. Binomial distribution

$n = 10,\ p = 0.2,\ q = 0.8.$

(1) $P(X=1) = \binom{10}{1}(0.2)(0.8)^9$

$= 10 \times 0.2 \times 0.1342$

$= 0.2684$

(2) $P(X>2) = 1 - [P(X=0) + P(X=1) + P(X=2)]$

$= 1 - \left[(0.8)^{10} + 0.2684 + \frac{10.9}{2.1}(0.2)^2(0.8)^8\right]$

$= 1 - [0.1074 + 0.2684 + 0.3020]$

$= 0.322$

(3) $\mu = np = 10 \times 0.2 = 2$

b. Poisson distribution, mean = 15

$P(X=10) = \frac{e^{-15}\, 15^{10}}{10!} = 0.0486.$

Q9. a. Assume that the lifetime of each tube is $L_i \sim N(\mu, \sigma^2)$

then $\sum_{i=1}^{n} L_i \sim N(n\mu, n\sigma^2)$

In this case $n = 3$ and if $L = L_1 + L_2 + L_3$,

$L \sim N(3\mu, 3\sigma^2) = N(\mu_L, \sigma_L^2)$

So $\mu_L = 3 \times 1500 = 4500h$.

$\sigma_L^2 = 3 \times 150^2$, $\therefore \sigma_L \doteqdot 260h$.

So $P(L > 5000) = P\left(Z > \frac{5000 - 4500}{260}\right)$

$= P(Z > 1.92)$

$= 0.0274$

Also $P(L < 4200) = P(Z < -1.15)$

$= 0.1251$

Q. 9. b. Specimen test: normal stress vs. shear resistance

x_i	y_i	$(x_i-\bar{x})$	$(y_i-\bar{y})$	$(x_i-\bar{x})(y_i-\bar{y})$	$(x_i-\bar{x})^2$	$(y_i-\bar{y})^2$
27.4	21.4					
26.8	26.5					
25.6	24.9					
25.4	27.3					
28.9	24.2					
23.6	27.1					
27.7	23.6					
23.9	25.9					
24.7	26.3					
28.1	22.5					
26.9	21.7					
22.6	25.8					
311.6	297.2	0	0	−29.533	43.045	47.146

$\bar{x} = 25.967, \ \bar{y} = 24.767$

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\left\{ \sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2 \right\}^{1/2}} = \frac{-29.533}{\left\{ 43.045 \times 47.146 \right\}^{1/2}} = -0.6556$$

To test $H_0 : \rho_0 = -0.5$ vs. $H_1 : \rho_0 \neq 0.5$ at the 5% level

$$Z = \frac{\sqrt{n-3}}{2} \left[\ln\left(\frac{1+r}{1-r}\right) - \ln\left(\frac{1+\rho_0}{1-\rho_0}\right) \right]$$

$$= 1.5 \ln\left(\frac{0.2080}{0.3333}\right) = -0.7074$$

This satisfies $|Z| \leq 1.96$, so accept H_0.