EMAT 20200
APPLIED STATISTICS
SOLUTIONS

2. PROBABILITY DISTRIBUTIONS – Properties:

Poisson Approximation to the Binomial Distribution Solution:

Binomial P[X = 0] = ${}^{100}C_0\ 0.05^0(1\text{-}0.05)^{100}$ = 0.3660.

Poisson P[X = 0] = $\dfrac{e^{-m}m^x}{x!}$ = 0.3679.

In practice the approximation is good for small n provided $X << n$.

Poisson Processes Solution:

This is a Poisson process

(a) Let X be the number of failures in 400 hours, then $E[X]$ and $P(X = x)$ are

$$E[X] = \frac{400}{200} = 2,$$

$$\therefore P(X = x) = e^{-2}\frac{2^x}{x!}$$

$$\therefore P(X = 0) = e^{-2} = 0.1353$$

(b) Let Y be the number of failures in 1000 hours.

$$E[Y] = \frac{1000}{200} = 5$$

$$P(Y = y) = e^{-5}\frac{5^y}{y!}$$

P($Y \geq 2$) = 1-[P($Y=0$)+P($Y=1$)] since events are independent

= 1-{e^{-5} + $5e^{-5}$}

= 0.9596

Distribution of the Time between Events in a Poisson Process Solution:

P(A) = 0.9596

P(B) = P(Y³$4$)

= 1 – [P(Y=0) + P(Y=1) +P(Y=2) + P(Y=3)]

$= 1 - [e^{-5} + 5e^{-5} + \frac{25}{2}e^{-5} + \frac{125}{6}e^{-5}]$

= 1 – 0.2650

= 0.7350

$$\therefore P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{P(Y \geq 2 \cap Y \geq 4)}{P(Y \geq 2)}$$

$$= \frac{P(Y \geq 4)}{P(Y \geq 2)} = \frac{0.7350}{0.9596}$$

$$= 0.7659 \left[\text{note that } A \subset B\right]$$

Calculation of probabilities associated with the standard normal distribution Solution:

Example 1:

Using type (3) we get:

(a) P(0 < Z < 1) = P(0 < Z < ∞) – P(1 < Z < ∞)
= 0.5 – 0.1587
= 0.3413

(b) P(-2 < Z < 2) = 2P(0 < Z < 2)
= 2(P (0 < Z < ∞) – P(2 < Z < ∞))
= 2(0.5 – 0.02275)
= 2 x 0.47725
= 0.9545

(c) P(0.3 ≤ Z ≤3.2) = P(0.3 < Z < ∞) – P(3.2 < Z < ∞)
= 0.3821 – 0.00069
= 0.38141

(d) P($|Z|$ > 0.3) = P(Z > 0.3) + P(Z < -0.3)
= 2P(Z > 0.3)
= 0.7642

Normal Distribution with Mean = m, Standard deviation = s

Solution:

Example 2:

We are required to find P($-5 \le T \le 5$) where T is distributed normally with $m = 0$, s = 2.5.

Transform to the standard normal variable $Z = \dfrac{T - m}{s}$

Then $t_1 = -5$ gives $z_1 = \dfrac{-5-0}{2.5} = -2$

and $t_2 = 5$ gives $z_2 = \dfrac{5-0}{2.5} = +2$

We require P ($-2 \le Z \le +2$) = 2 x P($Z \le +2$)

=0.9544 from Tables

Example 3:

Let the random variable T represent the lifetime of a component. We need to find $P(T \ge 105)$.

Transform to the standard normal variable $Z = \dfrac{T-100}{5}$

Hence $P(T \ge 105) = P(Z \ge 1)$
$= 0.1587$

Let B be the event the part has lasted at least 90 hours, and A be the event the part lasts for at least 105 hours.

We need to calculate $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

Now $P(A \cap B)$ = P (part lasts at least 105 hours) = 0.1587
$P(B)$ = P (part lasts at least 90 hours)

$$= P\left(Z \ge \frac{90-100}{5}\right)$$
$$= P(Z \ge -2)$$
$$= 0.9772$$

$\therefore$ P (part lasts at least 105 hours/part lasted for 90 hours) $= \dfrac{0.1587}{0.9772}$

$= 0.1624$

Normal Approximation to the Binomial Distribution Solution:

Example 4:

(a) The area of the Binomial histogram corresponding to 50 heads is bounded by x = 49½ and x = 50½. Also $m = np = 50$, $s = \sqrt{npq} = 5$. The corresponding values of the standard Normal variable Z are

$$z_1 = \frac{49\frac{1}{2} - 50}{5} = -0.1 \qquad \text{and} \qquad z_2 = \frac{50\frac{1}{2} - 50}{5} = +0.1$$

$$\therefore P \text{ (50 heads)} = P(-0.1 \le Z \le 0.1)$$

$$= 0.0796$$

(b) The area of the Binomial histogram corresponding to more than 60 heads is bounded below by x = 60½.

The corresponding value of the standard normal variable Z is

$$z_1 = \frac{60\frac{1}{2} - 50}{5} = 2.1$$

$$\therefore P \text{ (more than 60 heads)} = P(Z \ge 2.1)$$

$$= 0.01786$$