Example Simpson's rule, 2 panels, 4 panels 66p0

$$\int_0^{\pi/2} f(x)\,dx \quad , \quad f(x) = \sin(x)$$

a) Here $h = \frac{1}{2}\left(\frac{\pi}{2} - 0\right) = \frac{\pi}{4}$.

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[f(a) + 4f(a+h) + f(b)\right]$$

$$= \frac{\pi}{12}\left[\sin(0) + 4\sin\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{2}\right)\right]$$

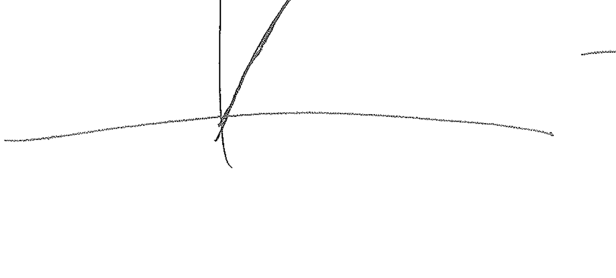
$= \underline{1.0023}$ Compare: 0.785 - one panel before

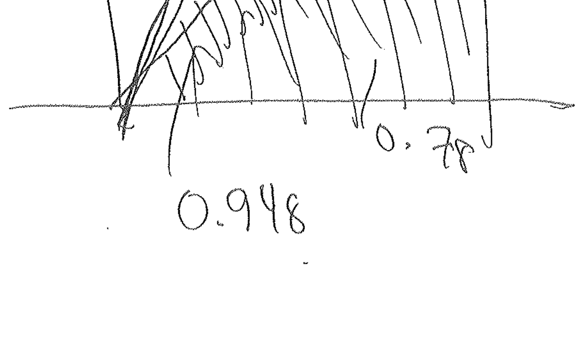
0.948 - 2

b) 4 panels $h = \frac{1}{4}\left(\frac{\pi}{2}\right) = \frac{\pi}{8}$

$$\int_a^b f(x)\,dx = \frac{\pi}{24}\left(\sin(0) + 4\sin\left(\frac{\pi}{8}\right) + 2\sin\left(\frac{\pi}{4}\right) + 4\sin\left(\frac{3\pi}{8}\right) + \sin\left(\frac{\pi}{2}\right)\right)$$

$= \underline{1.0001}$. = GOOD.





L6 ODE schemes

Use Euler's method to solve $\frac{dx}{dt} = x + \sin(t)$ $\quad x(0)=1$

get approx to $x(0.3)$. use $h = 0.1$.

$x_{n+1} = x_n + h\, f(x_n, t_n)$ | here $f(x,t) = x + \sin t$

$t_0 = 0$, $x_0 = 1$

$t_1 = t_0 + 1h = 0.1$, $\quad x_1 = x_0 + h(x_0 + \sin(t_0))$

$= 1 + 0.1(1 + \sin(0))$

$= 1.1$

$t_2 = t_0 + 2h = 0.2 \quad x_2 = x_1 + h(x_1 + \sin(t_1))$

$= 1.1 + 0.1(1.1 + \sin(0.1))$

$= 1.2199$

$t_3 = 0.3 \quad x_3 = x_2 + h(x_2 + \sin(t_2))$

$= 1.3618$

Use Euler method to solve

$$\frac{dx}{dt} = 4x - x^3 \qquad x(0) = 1$$

$x_0 = 1$

$t_0 = 0$

Same idea

$x_1 = x_0 + h f(x_0) \Rightarrow x_1 = 1 + 0.5(4(1) - 1^3)$
$= 2.5$

$x_2 = x_1 + h f(x_1)$ $\qquad x_2 = 2.5 + 0.5(4(2.5) - 2.5^3)$
$= -0.3125$

$x_3 = x_2 + h f(x_2)$ $\qquad x_3 = -0.92$

$\vdots$

$x_9 = 2.33$

↓

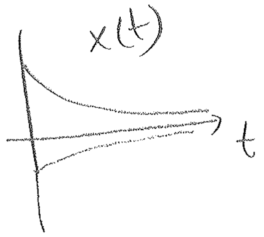
BADNESS!

$$\frac{dx}{dt} = \lambda x \quad \rightarrow \quad \frac{dx}{x} = \lambda dt$$

$$\ln x = \lambda t + C$$

$$x = A e^{\lambda t}$$

For $\lambda < 0$, $x(t)$ should decay:



Stability properties of the midpoint rule

$$\frac{x_{n+1} - x_{n-1}}{2h} = f(x_n)$$

$$\frac{x_{n+1} - x_{n-1}}{2h} = \lambda x_n$$

Suppose $f(x_n) = \lambda x_n$.

$$x_{n+1} - 2h\lambda x_n - x_{n-1} = 0, \text{ write } k = n-1$$

Sub $x_k = c\mu^k$, ~~...~~, $x_{k+2} - 2h\lambda x_{k+1} - x_k = 0$

get $\mu^2 - 2h\lambda\mu - 1 = 0$

$$\mu = \frac{1}{2}\left(2h\lambda \pm \sqrt{4h^2\lambda^2 + 4}\right) = h\lambda \pm \sqrt{h^2\lambda^2 + 1}$$

decaying if $\mu < 1$. Recall $\lambda < 0$: ~~...~~ and solution should decay.

$h\lambda + \sqrt{h^2\lambda^2 + 1} < 1$. Turns out always < 1 for $\lambda < 0$.

Exercise: show that $h\lambda + \sqrt{h^2\lambda^2 + 1} < 1$ when $\lambda < 0$.