SECTION A

Q1 A primitive binary receiver has probability p of making an error in receiving each bit of a transmitted message, i.e. there is a probability p that a '1' will be received as a '0', and vice-versa. To overcome this '111' is transmitted instead of '1', and '000' instead of '0'. The triple bits '011', '101', '110', as received, are interpreted as '1', and '001', '010', '100' as '0'.

(a) Show that the probability that a triple bit transmission is correctly received is:

$$F(p) = (1-p)^3 + 3p(1-p)^2$$

How does this accuracy compare for $p = 0.01$ and 0.05.

(b) Explain why a double bit transmission, e.g. '11' for '1' for accuracy improvement, would be totally useless.

(8 marks)

'111' is received when '111' is transmitted with probability $(1-p)^3$ — allows for 'p' error in each.

'011' is received with probability $p(1-p)^2$

Also for '101' and '110', we have probability $p(1-p)^2$.

Total: $(1-p)^3 + 3p(1-p)^2$.

p	0.01	0.05
	0.9997	0.9928.

'11' — ok.

'01' or '10' ?

00

SECTION A

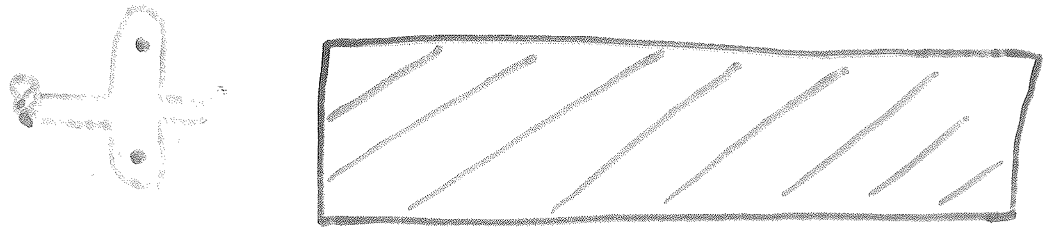
Q1 (a) In attempting to find a lost boat in bad weather a search aircraft is given only a 20% chance of detecting the boat in a random sweep over the sea nearby. Two sightings in two separate sweeps are deemed necessary before a lifeboat can be committed to the search. Show that 18 sweeps are necessary to ensure a 90% probability that the lifeboat will put to sea.

(5 marks)

(b) In estimating the true mean μ of a population a large sample ($n>30$) can be taken to determine the sample parameters $\bar{x}$ and s. For the sample size $n=390$, $|\bar{x}-\mu|$ can be estimated to within 0.1σ with 95% confidence. What sample size is needed to estimate $|\bar{x}-\mu|$ to within 0.01σ with the same confidence?

(3 marks)

Q1.a. Search sweeps over the sea are usually rectangular.



$P(\text{success}) = 0.2. \quad q = 0.8$

If X denotes the number of sightings, and n the number of sweeps (trials), the we require $P(X \geq 2)$ in n trials.

For n sweeps $P(X=0) = (0.8)^n$

$$P(X=1) = n(0.8)^{n-1}(0.2)$$

$$\therefore P(X < 2) = (0.8)^{n-1}(0.8 + n \times 0.2)$$

For $n = 18$, we have

$$(0.8)^{17} \times 4.4 = 0.09908$$

and $n = 17$,

$$(0.8)^{16} \times 4.2 = 0.11822$$

So $n = 18$ is needed to ensure

$$P(X < 2) < 0.1.$$

Q8 The Home Office has asked the police to investigate a suspected high incidence of illegal road use in the Bristol area. Government statisticians estimate that 20% of vehicles on the road in the Bristol area are illegal.

(a) The police are to conduct a random sample of 10 vehicles. Use a suitable discrete distribution to answer the following:

(i) What is the probability of just one illegal vehicle in the sample?

(2 marks)

(ii) What is the probability of more than two illegal vehicles in the sample?

(2 marks)

(iii) Determine the average number of illegal vehicles the police can expect in this sample.

(1 mark)

(b) The police are to carry out the sample along a stretch of road where the average number of vehicles per hour is 15. Due to manpower constraints the time allocated to the sample is one hour. Assuming random arrivals use a suitable discrete distribution to determine the probability of exactly ten vehicles arriving in the hour the sample is conducted.

(3 marks)

(c) The time between the arrival of vehicles is a continuous random variable, T, measured in minutes, with a probability density function, $f_T(t)$, and a cumulative distribution function, $F_T(t)$, as described below:

$$f_T(t) = \begin{cases} 0.25e^{-0.25t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$F_T(t) = \begin{cases} 1 - e^{-0.25t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

(i) Sketch $f_T(t)$ and $F_T(t)$

(4 marks)

(ii) Find the probability of the police having to wait more than 6 minutes for a vehicle to arrive.

(3 marks)

(iii) Prove that the expected time between arrivals is 4 minutes.

(5 marks)

Q. 8

a. Binomial distribution

$n = 10,\ p = 0.2,\ q = 0.8.$

(1) $P(X=1) = \binom{10}{1}(0.2)(0.8)^9$

$= 10 \times 0.2 \times 0.1342$

$= 0.2684$

(2) $P(X>2) = 1 - [P(X=0) + P(X=1) + P(X=2)]$

$= 1 - \left[(0.8)^{10} + 0.2684 + \frac{10.9}{2.1}(0.2)^2(0.8)^8\right]$

$= 1 - [0.1074 + 0.2684 + 0.3020]$

$= 0.322$

(3) $\mu = np = 10 \times 0.2 = 2$

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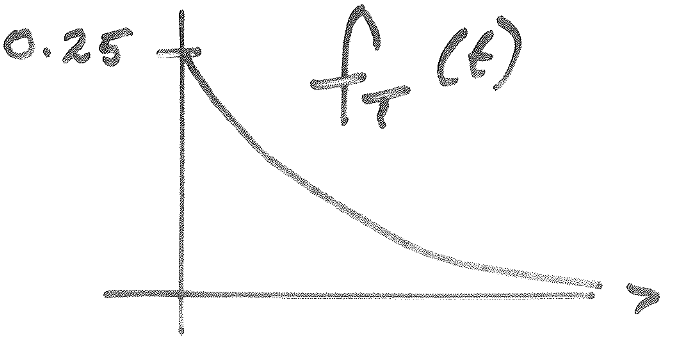
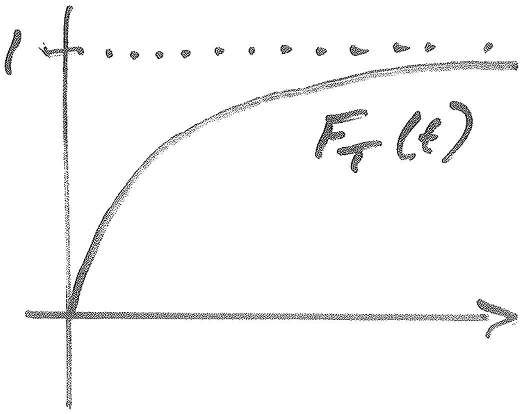
b. Poisson distribution, mean $= 15$

$P(X=10) = \frac{e^{-15}\, 15^{10}}{10!} = 0.0486.$

6. Negative exponential distribution

(1)





(2) $$P(T>6) = 1 - P(T \leq 6)$$
$$= 1 - F_T(6)$$
$$= 1 - (1 - e^{-6/4}) = e^{-1.5}$$
$$= 0.223$$

(3) $$E[T] = \int_{-\infty}^{\infty} t\, f_T(t)\, dt$$
$$= \int_0^{\infty} t \cdot \frac{1}{4} \cdot e^{-t/4}\, dt$$
$$= \left[-t e^{-t/4}\right]_0^{\infty} + \int_0^{\infty} e^{-t/4}\, dt$$
$$0$$
$$= \left[-4 e^{-t/4}\right]_0^{\infty} = 4.$$

Q8 (a) A random variable X follows a Poisson distribution of mean μ and satisfies

$$P(X = x) = \frac{\mu^x e^{-\mu}}{x!}; \quad 0! = 1.$$

Show that

$$P(X = x+1) = \frac{\mu}{x+1} \cdot P(X = x)$$

(4 marks)

(b) Radioactive emissions are observed over 400 one-millisecond intervals and tabulated accordingly.

Number of emissions	Number of intervals
0	3
1	15
2	47
3	76
4	68
5	74
6	46
7	39
8	15
9	9
10	5
11	2
12	0
13	1

Test the Null hypothesis that the observations follow a Poisson distribution with $\mu = 4.6$. Use a 5% level of significance.

(16 marks)

EXTRA Q

turn over / ...

Q No.	PAPER NAME .... Q. SETTER INITIALS MI/JB	No. of Marks

Q8 (a) For the Poisson distribution,

$$P(X=x) = \frac{\mu^x e^{-\mu}}{x!}, \quad X = 0, 1, 2 \ldots$$

Assuming $0! = 1$, then

$$P(X=0) = e^{-\mu}. \quad (2)$$

Now $P(X=x+1) = \frac{\mu^{x+1} e^{-\mu}}{(x+1)!} = \frac{\mu}{x+1} \cdot \frac{\mu^x e^{-\mu}}{x!}$

$$= \frac{\mu}{x+1} \cdot P(X=x) \quad (2)$$

4

(b) Noting the observed frequencies of 400 radioactive emission observations taken over 1 millisecond intervals, and comparing them to expected frequencies based on a Poisson distribution with $\mu = 4.6$, we get

No of emissions	'O'	Poisson 'prob'	'E'	
0	3	0.010	4.0	} 22.4
1	15	.046	18.4	
2	47	.107	42.8	
3	76	.163	65.2	
4	68	.187	74.8	
5	74	.173	69.2	
6	46	.132	52.8	
7	39	.087	34.8	
8	15	.050	20.0	
9	9	.025	10.0	
10	5	.012	4.8	} 8.0
11	2	.005	2.0	
12	0	.002	0.8	
13	1	.001	0.4	(8)

Q No.	PAPER NAME .... Q. SETTER INITIALS MDJB	No. of Marks
Q8 Cont.	Regrouping of the expected frequencies is necessary as $E < 5$ at either end. (2)	

There remain 10 class intervals corresponding to 1-10 emissions. The number of degrees of freedom is 10-1 (not 10-2) as we are testing against a Poisson distribution of defined mean, not one whose mean arises from the observations.

We calculate

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$= \frac{(18-22.4)^2}{22.4} + \frac{(47-42.8)^2}{42.8} + \cdots \frac{(9-10.0)^2}{10} + \frac{(8-8.0)^2}{8}$$

$= 6.749.$ (4)

Note that were asked to test

$H_0: \mu = \mu_0 \quad (4.6)$ .vs.

$H_1: \mu \neq \mu_0$

Since $\chi^2_{,0.05;9} = 16.919$, we accept H_0, i.e. the fit is good at the 5% level. (2)

16

20

Q9 (a) A 20 litre drum of creosote covers an average 513.3m^2 of fencing, with standard deviation 31.5m^2. What is the probability that the sample mean area covered by a sample of size 40 of these drums will be between 510m^2 to 520m^2.

(6 marks)

(b) The data below are measurements of the air velocity and evaporation coefficient of burning fuel droplets in a jet engine.

Air Velocity (cm s^{-1}) x	Evaporation Coefficient (mm^2s^{-1}) y
20	0.18
60	0.37
100	0.35
140	0.78
180	0.56
220	0.75
260	1.18
300	1.36
340	1.17
380	1.65

To fit a line of regression of y on x, i.e. $y = a + bx$, we can determine a and b from the Normal Equations,

$$\sum_{i=1}^{n} y_i = an + b\sum_{i=1}^{n} x_i$$

$$\sum_{i=1}^{n} x_i y_i = a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} x_i^2$$

Use these to show that

$$y = 0.069 + 0.00383x$$

and determine the value of the evaporation coefficient when the air velocity is 190 cm s^{-1}.

(14 marks)

EXTRA Q

Q No.	PAPER NAME .... Q. SETTER INITIALS MDJB	No. of Marks
Q9	(a) Given that a 20 litre drum of creosote covers on average $513.3\ m^2$ of fencing with standard deviation $31.5\ m^2$, a sample of 40 such cans will have its mean, $\bar{x}$, distributed $N\left(513.3, \frac{(31.5)^2}{40}\right)$. (2)	
	So $P(510.0 < \bar{x} < 520.0)$	
	$= P\left(\frac{510-513.3}{31.5/\sqrt{40}} < Z < \frac{520-513.3}{31.5/\sqrt{40}}\right)$ (2)	
	$= P(-0.66 < Z < 1.34) = 0.6553$, where $Z \sim N(0,1)$. (2)	6
	(b) With the data	

Air velocity	Evaporation Coefficient
20	0.18
60	0.37
100	0.35
140	0.78
180	0.56
220	0.75
260	1.18
300	1.36
340	1.17
380	1.65

Q No.	PAPER NAME .... Q. SETTER INITIALS MDJB	No. of Marks
Q9 Cont.	We have, $n = 10$,	
	$\sum x_i = 2000 \quad \sum x_i^2 = 532{,}000$	
	$\sum y_i = 8.35 \quad \sum x_i y_i = 2{,}175.40$	
	$\sum y_i^2 = 9.1097$ (6)	
	The Normal Equations thus give	
	$8.35 = 10a + 2000b$	
	$2{,}175.40 = 2000a + 532{,}000b$ (2)	
	These solve to give –	
	$a = 0.069, \; b = 0.00383,$	
	so the line of regression of y on x is:	
	$y = 0.069 + 0.00383x$ (4)	
	For the case of $x = 190 \text{ cm s}^{-1}$,	
	$y = 0.069 + 0.00383(190)$	
	$= 0.80 \text{ mm}^2 \text{s}^{-1}$. (2)	14
		20