Answers

Marks: [x]

1 Calculate grad f where $f = 3x^2 + 2xy + 3zx^3$. Calculate the directional derivative at the point $(x, y, z) = (2, 3, 4)$ in the direction of the vector $(1, 1, 1)$.

$$\nabla f = (f_x, f_y, f_z) = (6x + 2y + 9x^2 z, 2x, 3x^3) \quad [2]$$

$$D_a f(r_0) = \nabla f \cdot \hat{a}\big|_{r=r_0} \qquad a = (1,1,1);\ |a| = \sqrt{3}$$

$$\Rightarrow \hat{a} = \frac{1}{\sqrt{3}}(1,1,1) \quad [1]$$

$$\nabla f\big|_{(2,3,4)} = (12 + 6 + 9 \times 16, 4, 24) = (162, 4, 24) \quad [1]$$

$$\Rightarrow D_a f(r_0) = (162, 4, 24) \cdot (1,1,1)/\sqrt{3}$$

$$= 190/\sqrt{3} \quad [1]$$

2 Determine whether the unique stationary point at the origin is a maximum minimum or saddle for the function

$$\phi(x, y, z) = x^2 + xy + 2y^2 + z^2 = 0$$

$$\phi_x = 2x + y \ ; \ \phi_y = x + 4y \ , \ \phi_z = 2z$$

$$\phi_{xx} = 2 \ ; \ \phi_{xy} = 1 \ ; \ \phi_{yy} = 4 \ , \ \phi_{zz} = 2$$

ϕ''_{yx} all other derivatives $= 0$.

$$\Rightarrow H = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad [2]$$

eigenvalues: $\lambda = 2$ and ev of $\begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}$

[1]

$$\begin{vmatrix} 2-\lambda & 1 \\ 1 & 4-\lambda \end{vmatrix} = 7 - 6\lambda + \lambda^2 \Rightarrow \lambda = 3 \pm \frac{\sqrt{36-28}}{2}$$

$$= 3 \pm \sqrt{2} \quad [1]$$

$\Rightarrow$ 3 positive eigenvalues

$\Rightarrow$ minimum point [1]

3 For the vector field $\boldsymbol{v} = x^3\boldsymbol{i} + y^2x\boldsymbol{j} + yz^3\boldsymbol{k}$ calculate div $\boldsymbol{v}$, curl $\boldsymbol{v}$ and div curl $\boldsymbol{v}$.

$$\operatorname{div} V = \frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial y} = 3x^2 + 2xy + 3yz^2 \quad [2]$$

$$\operatorname{curl} v = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^3 & y^2x & yz^3 \end{vmatrix} = \begin{pmatrix} z^3 - 0 \\ -(0-0) \\ y^2 - 0 \end{pmatrix} = \begin{pmatrix} z^3 \\ 0 \\ y^2 \end{pmatrix} \quad [2]$$

$$(\text{or } z^3 i + y^2 k)$$

<u>always:</u> $\operatorname{div} \operatorname{curl} v = 0$ [1]