

Name:

Degree Course:

Student number:

Name of Tutor:

Name of marker:

The question is worth 10 marks. There are 15 minutes available.

All working should be contained on this paper

No written material (course notes, textbooks etc.) is to be consulted during the test.

- 1 Using Stoke's Theorem, calculate the integral  $\int_C \mathbf{F} \cdot d\mathbf{r}$  for the vector field  
 $\mathbf{F} = (2x + y^2)\mathbf{i} + (2xy + z^2)\mathbf{j} + (2yz + 2)\mathbf{k}$ ,  
 where  $C$  is the closed circle given by  $x^2 + y^2 = 4$  and  $z = 2$ .

Answer:

Stokes Theorem:

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \text{curl } \mathbf{F} dA \quad [2]$$

$$\text{curl } \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2x+y^2 & 2xy+z^2 & 2zy+2 \end{vmatrix} \quad [2]$$

$$= \begin{pmatrix} 2z - 2z \\ 0 - 0 \\ 2y - 2y \end{pmatrix} = \mathbf{0} \quad [3]$$

$$\Rightarrow \iint_S \text{curl } \mathbf{F} dA = 0 \quad [3]$$